Estimation of Stress Strength Reliability in Single Component Models for Different Distributions

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Authors’ contributions

This work was carried out in collaboration among all authors. Authors NB and TRJ designed the study, performed the statistical analysis, wrote the protocol and wrote the first draft of the manuscript. Authors RB and SAM managed the analyses of the study and literature searches. All authors read and approved the final manuscript.

Article Information

DOI: 10.9734/ CJAST/2019/v34i630155  
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(2) Prasanth Warrier, Calicut University, India.  
Yahaya Shagaiya Daniel, Kaduna State University, Nigeria.  
Complete Peer review History: http://www.sdiarticle3.com/review-history/48629

ABSTRACT

This paper aims to estimate the stress-strength reliability parameter \( R = P(Y < X) \), considering the two different cases of stress strength parameters, when the strength ‘\( X \)’ follows exponentiated inverse power Lindley distribution, extended inverse Lindley and Stress ‘\( Y \)’ follows inverse power Lindley distribution and inverse Lindley distribution. The method of maximum likelihood estimation is used to obtain the reliability estimators. Illustrations are provided using R programming.

Keywords: Lindley distribution (LD); Inverse Lindley distribution (ILD); Inverse Power Lindley distribution (IPLD); Extended Inverse Lindley distribution (EILD); Exponentiated Inverse Power Lindley distribution (EIPLD); Maximum likelihood estimator (MLE).

1. INTRODUCTION

The LD was first introduced by D. V. Lindley [1]. The distribution is a mixture of the gamma distribution, with shape parameter 2 and scale parameter \( \beta \) and exponential distribution with scale parameter \( \beta \). Its probability distribution function (pdf) is given by

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The analogous cumulative rate data. The upside inverted version of the LD that can be used to model the upside bathtub shape hazard rates, V.

Cancer machines, survival of head and neck cancer rate function that is efficient of modeling many models with upside bathtub shape hazard.

V. K. Sharma and Khandelwal [10], known as EILD which deals with more malleability with the effective shape parameter. Its probability density function (pdf) is given by:

\[ f(x; \alpha, \beta) = \frac{\beta^\alpha}{(1 + \beta)^{\alpha+1}} \cdot \left( 1 + \frac{x}{\beta} \right)^{-\alpha-1} e^{-\frac{\beta}{x}} \cdot x > 0, \beta > 0, \alpha > 1. \]

In order to accomplish more flexible family of distributions, another generalization is the IPLD suggested by Barco, Mazucheli and Janeiro [9] by considering the power transformation, \[ X = Y^{-\alpha}. \] Explicitly if a random variable \( Y \) follows ILD, then the random variable \( Z = Y^{\alpha} \) follows IPLD with density and cumulative distribution functions defined respectively as

\[ f(z; \alpha, \beta) = \frac{a\beta^2}{1 + \beta} \cdot \left( 1 + \frac{z^\alpha}{\beta} \right)^{-\frac{1}{\alpha}} \cdot z > 0, \alpha > 0, \beta > 0. \]

\[ F(z; \alpha, \beta) = \left( 1 + \frac{\beta}{1 + \beta} z^\alpha \right)^{-\frac{1}{\alpha}} \cdot z > 0, \alpha > 0, \beta > 0. \]

The ILD take into account the inverse of a random variable with a LD. If a random variable \( Y \) has a LD, then a random variable \( Y = 1/X \) follows ILD with probability distribution function defined by

\[ f(y; \alpha, \beta) = \frac{\beta^\alpha}{1 + \beta} \cdot \left( 1 + \frac{x}{\beta} \right)^{-\alpha-1} e^{-\frac{\beta}{x}} \cdot x > 0, \beta > 0. \]

The corresponding cumulative distribution function (cdf) is given as:

\[ F(y; \alpha, \beta) = \left( 1 + \frac{\beta}{1 + \beta} x \right)^{-\frac{1}{\alpha}} \cdot y > 0, \alpha > 0, \beta > 0. \]

In the above cited reference, the authors mainly fixate on the estimation of increasing, decreasing and bathtub shaped failure rate function. V.K. Sharma, S. K. Singh and U. Singh [7] proposed a lifetime model with upside-down bathtub shape hazard rate function that is efficient of modeling many real problems, for example failure of washing machines, survival of head and neck cancer patients, and survival of patients with breast cancer. Considering the fact that all inverse distribution acquire the upside-down bathtub shape for their hazard rates, V.K. Sharma, S. K. Singh, U. Singh and V. Agiwale [8], proffered an inverted version of the LD that can be used to model the upside-down bathtub shape hazard rate data.

The analogous cumulative distribution function (cdf) is given by:

\[ F(x; \alpha, \beta) = pCDFInverseGamma(\alpha - 1, \beta) + (1 - p)CDFInverseGamma(\alpha, \beta) \cdot \beta > 0. \]
Where, $CDF_{\text{InverseGamma}}(\alpha, \beta) = \frac{\Gamma(\beta / x, \alpha)}{\Gamma \alpha}$, $\Gamma(\alpha, \alpha) = \int_x^\infty e^{-t} \, dt$ and $\Gamma \alpha = \int_0^1 e^{-t} \, dt$.

A new three parameter probability distribution introduced by R. Jan et al. [11] known as Exponentiated inverse power Lindley distributed (EIPLD). Its pdf and cdf is given by:

$$f(x) = \frac{\alpha \beta^2 \gamma}{1 + \beta} \left(1 + x^\alpha\right)^{-\gamma} \left(1 + \frac{\beta}{1 + \beta} x^\alpha\right)^{-\gamma-1} x > 0, \; \alpha, \beta, \gamma > 0$$

$$F(x) = \left[\left(1 + \frac{\beta}{1 + \beta} x^\alpha\right)^{-\gamma}\right] x > 0, \; \alpha, \beta, \gamma > 0$$

The stress strength parameter plays an important role in the reliability analysis (Fig. 2). For example if $X$ is the strength of a system which is subjected to stress $Y$, then the parameter $R = P(Y < X)$ measures the system performance and it is very common in the context of mechanical reliability of a system. Moreover, $R$ provides the probability of a system failure, if the system fails whenever the applied stress is greater than its strength. Many authors developed the estimation procedures for estimating the stress–strength reliability from various lifetime models, see [12,13] and references cited therein. Recently, M. M. Mohie El-Din, A. Sadek and Sh. H. Elmegahwry [14] obtained Stress–Strength Reliability Estimation for Exponentiated Generalized Inverse Weibull Distribution. T. R. Rasethuntsa and M. Nadar [15] derived Stress–strength reliability of a non-identical-component-strengths system based on upper record values from the family of Kumaraswamy generalized distributions. A. Iranmanesh, K. F. Vajargah and M. Hasanzadeh [16] studied estimation of stress strength reliability parameter of inverted gamma distribution.

In this paper, we have addressed the problem of estimating $R = P(Y < X)$ considering the two different cases for stress strength reliability

1) When stress follows IPLD and strength follows EIPLD.
2) When stress follows ILD and strength follows EILD.

2. RELIABILITY AND ITS MAXIMUM LIKELIHOOD FUNCTION

CASE 1: Let $Y \sim \text{IPLD}(\alpha, \beta_1)$ and $X \sim \text{EIPLD}(\alpha, \beta_2, \gamma)$ be independent random variables,

Suppose that $X$ represent the strength of a component exposed to $Y$ stress, then the stress strength reliability(SSR) of this component is obtained as follows,

$$R = \Pr(X > Y) = \int_0^\infty p(X > Y / Y = y) f_Y(y) \, dy$$

$$= \int_0^\infty S_Y(y) f_Y(y) \, dy$$

$$= \int_0^\infty \left(1 + \frac{\beta_1}{1 + \beta_1} e^{-\frac{\beta_1}{1 + \beta_1}}\right)^\gamma \left(1 + x^\alpha\right)^{-\gamma} \left(1 + \frac{\beta}{1 + \beta} x^\alpha\right)^{-\gamma-1} x > 0, \; \alpha, \beta, \gamma > 0$$

$$= \frac{\alpha \beta^2 \gamma}{1 + \beta_2} \int_0^\infty \left(1 + \frac{\beta_1}{1 + \beta_1} e^{-\frac{\beta_1}{1 + \beta_1}}\right)^\gamma \left(1 + x^\alpha\right)^{-\gamma} \left(1 + \frac{\beta}{1 + \beta} x^\alpha\right)^{-\gamma-1} x > 0, \; \alpha, \beta, \gamma > 0$$

$$= \frac{\alpha \beta^2 \gamma}{1 + \beta_2} \int_0^\infty \left(1 + \frac{\beta_1}{1 + \beta_1} e^{-\frac{\beta_1}{1 + \beta_1}}\right)^\gamma \left(1 + x^\alpha\right)^{-\gamma} \left(1 + \frac{\beta}{1 + \beta} x^\alpha\right)^{-\gamma-1} x > 0, \; \alpha, \beta, \gamma > 0$$

$$= \frac{\alpha \beta^2 \gamma}{1 + \beta_2} \int_0^\infty \sum_{i=1}^{\infty} C_i \left(\frac{\beta_1}{1 + \beta_1}\right)^i \left(1 + x^\alpha\right)^{-\gamma} \left(1 + \frac{\beta}{1 + \beta} x^\alpha\right)^{-\gamma-1} x > 0, \; \alpha, \beta, \gamma > 0$$

$$= \frac{\alpha \beta^2 \gamma}{1 + \beta_2} \int_0^\infty \sum_{i=1}^{\infty} C_i \left(\frac{\beta_1}{1 + \beta_1}\right)^i \left(1 + x^\alpha\right)^{-\gamma} \left(1 + \frac{\beta}{1 + \beta} x^\alpha\right)^{-\gamma-1} x > 0, \; \alpha, \beta, \gamma > 0$$

$$= \frac{\alpha \beta^2 \gamma}{1 + \beta_2} \int_0^\infty \sum_{i=1}^{\infty} C_i \left(\frac{\beta_1}{1 + \beta_1}\right)^i \left(1 + x^\alpha\right)^{-\gamma} \left(1 + \frac{\beta}{1 + \beta} x^\alpha\right)^{-\gamma-1} x > 0, \; \alpha, \beta, \gamma > 0$$

$$= \frac{\alpha \beta^2 \gamma}{1 + \beta_2} \int_0^\infty \sum_{i=1}^{\infty} C_i \left(\frac{\beta_1}{1 + \beta_1}\right)^i \left(1 + x^\alpha\right)^{-\gamma} \left(1 + \frac{\beta}{1 + \beta} x^\alpha\right)^{-\gamma-1} x > 0, \; \alpha, \beta, \gamma > 0$$
\[
\begin{align*}
M = \beta_2^2 \sum_{i=1}^{\infty} \gamma C_i \left( \left[ \int_0^t e^{-t(\beta_1 + \beta_2)} dt \right]^{i+1} - \int_0^t e^{-t(\beta_1 + \beta_2)} dt \right)
\end{align*}
\]

where \( R \) is independent of \( \alpha \)

Suppose \( x_1, x_2, \ldots, x_n \) is a random sample of size \( n_1 \) from EIPLD \((\alpha, \beta_1, \gamma)\) and \( y_1, y_2, \ldots, y_n \) is an independent random sample of size \( n_2 \) from IPL \((\alpha, \beta_2)\). The likelihood function \( l = l(\theta) \) where \( \theta = (\alpha, \beta_1, \beta_2, \gamma) \) based on the two independent random sample is given by:

\[
l = \sum_{i=1}^{n_1} \ln(f_i(x_i)) + \sum_{j=1}^{n_2} \ln(f_j(y_j))
\]

\[
l = n_1 \ln(\alpha + 2\beta_1 + \ln y - \ln(1 + \beta_1)) + \sum_{i=1}^{n_1} \ln(1 + x_i^\beta) - (2\alpha + 1) \sum_{i=1}^{n_1} \ln x_i - \beta_1 \sum_{i=1}^{n_1} \frac{1}{x_i^\beta} + (y - 1) \sum_{j=1}^{n_2} \ln \left[ \frac{1}{1 + \beta_1} \right] + n_2 \left[ \ln \beta_2 - \ln(1 + \beta_2) \right] + \sum_{j=1}^{n_2} \ln(1 + y_j) - 3 \sum_{j=1}^{n_2} \ln(y_j) - \beta_2 \sum_{j=1}^{n_2} \frac{1}{y_j}
\]

The MLE \( \hat{\theta} \) of \( \theta \) is the solution of non-linear equations (1.2), (1.3), (1.4) (1.5)

\[
\frac{\partial l}{\partial \alpha} = \frac{n_1 + n_2}{\alpha} + \sum_{i=1}^{n_1} \beta_1 x_i^\alpha \ln x_i - 2 \sum_{i=1}^{n_1} \beta_1 y_j x_i^\alpha \ln x_i + (y - 1) \sum_{i=1}^{n_1} \frac{1}{1 + \beta_1} \ln \left( \frac{1}{x_i^\beta} \right) = 0
\]

\[
\frac{\partial l}{\partial \beta_1} = \frac{2n_1 + n_2 \beta_1}{\beta_1 (1 + \beta_1)} + \gamma \sum_{i=1}^{n_1} \frac{1}{x_i^\beta} + (y - 1) \sum_{i=1}^{n_1} \frac{x_i^\beta}{x_i^\beta + \beta_1} + \beta_1 \left( \frac{1}{\beta_1} + \frac{1}{1 + \beta_1} \right) = 0
\]

\[
\frac{\partial l}{\partial \beta_2} = \frac{2n_2 + n_2 \beta_2 - n_2 \beta_2}{\beta_2 (1 + \beta_2)} - \sum_{j=1}^{n_2} \frac{1}{y_j} = 0
\]

\[
= -\beta_2 \sum_{i=1}^{n_1} \frac{1}{y_j} - \beta_2 \left[ \sum_{j=1}^{n_2} \frac{1}{y_j} - n_2 \right] + 2n_2 = 0
\]

\[
\hat{\beta}_2 = \beta_2 (\hat{\alpha}) = \left( \sum_{j=1}^{n_2} \frac{1}{y_j} - n_2 \right) + \sqrt{\left( \sum_{j=1}^{n_2} \frac{1}{y_j} - n_2 \right)^2 + 8n_2} \sum_{j=1}^{n_2} \frac{1}{y_j}
\]

\[
\sum_{j=1}^{n_2} \left( \frac{1}{y_j} \right)
\]
\[
\frac{\partial l}{\partial \theta} = n_1 \frac{\partial}{\partial \theta} \left( -\beta_1 \sum_{i=1}^{n_1} x_i^{\alpha} + \sum_{i=1}^{n_1} \ln \left( 1 + \frac{\beta_1 \frac{1}{x_i^{\alpha}}}{1 + \beta_1 \frac{1}{x_i^{\alpha}}} \right) \right) = 0
\] (1.5)

\[
\hat{\theta} = \gamma(\hat{\alpha}) = \frac{n_1 \sum_{i=1}^{n_1} \frac{1}{x_i^{\alpha}} + \sum_{i=1}^{n_1} \ln \left( 1 + \frac{\beta_1 \frac{1}{x_i^{\alpha}}}{1 + \beta_1 \frac{1}{x_i^{\alpha}}} \right)}{\beta_1 \sum_{i=1}^{n_1} x_i^{\alpha} + \sum_{i=1}^{n_1} \ln \left( 1 + \frac{\beta_1 \frac{1}{x_i^{\alpha}}}{1 + \beta_1 \frac{1}{x_i^{\alpha}}} \right)}
\]

Where \( \hat{\alpha} \) is the solution of non linear equation:

\[
G(\alpha) = \frac{n_1}{\alpha} + \sum_{i=1}^{n_1} \frac{x_i^{\alpha} \ln x_i}{1 + x_i^{\alpha}} - 2 \sum_{i=1}^{n_1} \ln x_i + \beta_1 (\hat{\alpha}) \sum_{i=1}^{n_1} x_i^{\alpha} \ln x_i + (\gamma(\hat{\alpha}) - 1)(1 + \beta_1 (\hat{\alpha})) \sum_{i=1}^{n_1} \beta_1 (\hat{\alpha}) \ln \left( \frac{1}{x_i^{\alpha}} \right)
\]

Case 2: Let \( X \sim EILD(\alpha, \beta_1) \) and \( Y \sim ILD(\beta_2) \) be independent random variables, Suppose that \( X \) represent the strength of a component exposed to \( Y \) stress, then the Stress Strength Reliability (SSR) of this component is obtained as follows,

\[
R = P(X > Y) = \int_0^\infty p(X > Y | Y = y) f_Y(y) dy
\]

\[
R = \int_0^\infty \left( \int_0^\infty g(y) dy \right) f(x) dx
\]

\[
= \int_0^\infty \left( \int_0^\infty \frac{\beta_1}{1 + \beta_1} \frac{\beta_2}{1 + \beta_2} \left( 1 + \frac{1}{y^3} \right) e^{\frac{\beta_1}{y} x} \frac{\beta_1 e^{\frac{1}{y^3}} x^{(\alpha - 1) + 1}}{1 + \beta_1 x^{\alpha + 1} \Gamma(\alpha)} dx \right) dy
\]

\[
= \frac{\beta_1}{1 + \beta_1} \frac{\beta_2}{1 + \beta_2} \left( 1 + \frac{1}{y^3} \right) e^{\frac{\beta_1}{y} x} \left[ \frac{(\alpha - 1) e^{\frac{\beta_1}{y} x}}{\Gamma(\alpha)} dx + \frac{e^{\frac{\beta_1}{y} x}}{\Gamma(\alpha)} dx + (\alpha - 1) \frac{\beta_2}{(1 + \beta_2) (1 + \beta_2 + \frac{\beta_1}{\beta_2})} \right] \frac{e^{\frac{\beta_1}{y} x}}{\Gamma(\alpha + 1)} dx
\]

\[
R = \frac{\beta_1}{1 + \beta_1} \frac{\beta_2}{1 + \beta_2} \left( 1 + \frac{1}{y^3} \right) e^{\frac{\beta_1}{y} x} \left[ \frac{(\alpha - 1) e^{\frac{\beta_1}{y} x}}{\Gamma(\alpha)} dx + \frac{e^{\frac{\beta_1}{y} x}}{\Gamma(\alpha)} dx + (\alpha - 1) \frac{\beta_2}{(1 + \beta_2) (1 + \beta_2 + \frac{\beta_1}{\beta_2})} \right] \frac{e^{\frac{\beta_1}{y} x}}{\Gamma(\alpha + 1)} dx
\]

\[
(2.1)
\]

Suppose \( x_1, x_2, \ldots, x_n \) is a random sample of size \( n_1 \) from EILD \( (\alpha, \beta_1) \) and \( y_1, y_2, \ldots, y_n \) is an independent random sample of size \( n_2 \) from ILD \( (\beta_2) \). The likelihood function \( l = l(\theta) \) where \( \theta = (\alpha, \beta_1, \beta_2) \) based on the two independent random sample is given by:

\[
l = \sum_{i=1}^{n_1} \ln[f_X(x_i)] + \sum_{j=1}^{n_2} \ln[f_Y(y_j)]
\]
The MLE $\hat{\alpha}, \hat{\beta}_1, \hat{\beta}_2$ of $\alpha, \beta_1, \beta_2$ is the solution of non-linear equations (2.2), (2.3), (2.4)

\[
\frac{\partial}{\partial \alpha} l = n_1 \log \beta_1 + \sum_{i=1}^{n} \left( \frac{x_i}{(\alpha - 1) i} \right) - \sum_{i=1}^{n} \log x_i - \frac{n}{\Gamma} \frac{\partial}{\partial \alpha} \Gamma \alpha = 0
\]  
(2.2)

\[
\frac{\partial}{\partial \beta_1} l = n_1 \left( \frac{\alpha}{\beta_1} - \frac{1}{1 + \beta_1} \right) - \sum_{i=1}^{n} \frac{1}{x_i} = 0
\]

(2.3)

\[
\frac{\partial l}{\partial \beta_1} = \frac{n_1 \alpha - n_1 \alpha \beta_1 - n_1 \beta_1}{\beta_1 (1 + \beta_1)} - \sum_{i=1}^{n} \frac{1}{x_i} = 0
\]

\[
\hat{\beta}_1 = \beta_1(\hat{\alpha}) = \frac{\prod_{i=1}^{n} \frac{1}{x_i} - n_1 \alpha + n_1}{4n_1 \alpha \sum_{i=1}^{n} \frac{1}{x_i}}
\]

(2.4)

\[
\frac{\partial l}{\partial \beta_2} = \frac{2n_2}{\beta_2} - \frac{n_2}{1 + \beta_2} - \sum_{j=1}^{n} \left( \frac{1}{y_j} \right) = 0
\]

\[
\hat{\beta}_2 = \beta_2(\hat{\alpha}) = \frac{\prod_{j=1}^{n} \frac{1}{y_j} - n_2 + 2n_2}{8n_2 \sum_{j=1}^{n} \frac{1}{y_j}}
\]

Where $\hat{\alpha}$ is the solution of non-linear equation

\[
G(\alpha) = n_1 \log \beta_1(\hat{\alpha}) + \sum_{i=1}^{n} \left( \frac{x_i}{(\alpha - 1) i} \right) - \sum_{i=1}^{n} \log x_i - \frac{n}{\Gamma} \frac{\partial}{\partial \alpha} \Gamma \alpha
\]
### Table 1. Stress strength reliability when stress follows IPLD and strength follows EIPLD

<table>
<thead>
<tr>
<th>$\gamma = 1$</th>
<th>$\beta_2$</th>
<th>0.1</th>
<th>1.5</th>
<th>1.9</th>
<th>2.6</th>
<th>2.8</th>
<th>3.5</th>
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<tbody>
<tr>
<td>$\beta_1$</td>
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<td>0.0151</td>
<td>0.0097</td>
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### Table 2. Stress strength reliability when stress follows ILD and strength follows EILD

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<td>0.0097</td>
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Fig. 1.1. Graphical overview of stress-strength reliability expression (1.1) for gamma=1 and different values of beta 1 and beta 2

Fig. 1.2. Graphical overview of stress-strength reliability expression (1.1) for gamma=2 and different values of beta 1 and beta 2

Fig. 1.3. Graphical overview of stress-strength reliability expression (1.1) for gamma=3 and different values of beta 1 and beta 2
CONCLUSION

In this paper, we have studied the stress strength reliability considering the two different cases of stress strength parameters. When strength $X \sim EIPLD (\alpha, \beta_1, \gamma)$ and stress $Y \sim IPL (\alpha, \beta_1)$, it was observed that with increase in the value of strength parameter $\gamma$ with fixed parameters $(\beta_1, \beta_2)$, the stress strength reliability increases. However it is seen that, with increase in the value of stress parameter $\beta_2$, the stress strength reliability decreases keeping $(\gamma, \beta_1)$ fixed (Table 1). Further the graphical overview of Stress strength reliability for $\gamma_1, \gamma_2, \gamma_3$ for different values of $(\beta_1, \beta_2)$ are shown in Figs. (1.1, 1.2, 1.3) respectively. Also, when the strength $X \sim EILD(\alpha, \beta_1)$ and Stress $Y \sim ILD(\beta_2)$, it was found that as the value of stress parameter $\beta_2$ increases, keeping the strength parameters fixed $(\alpha, \beta_1)$, the stress strength parameter decreases. While as, with increase in the value of strength parameter $\beta_1$, the stress strength reliability increases, keeping $\alpha, \beta_2$ fixed (Table 2). Hence we conclude that with decrease in the value of stress parameter and increase in value of strength parameter, reliability of single component system increases resulting in efficiency of system model.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

REFERENCES


